Key

1. What is the instantaneous rate of change at $x = 3$ of the function f given by $f(x)=\dfrac{x^2-2x}{x+1}$?

$f'(x)=\dfrac{(x+1)(2x-2)-(x^2-2x)(1)}{(x+1)^2}$

$f'(3)=\dfrac{(4)(4)-3}{16}$

$\boxed{f'(3)=\dfrac{13}{16}}$

2. If a ≠ 0 then $\lim\limits_{x\to a}\dfrac{x-a}{x^3-a^3}=$ $\dfrac{a-a}{a^3-a^3}=\dfrac{0}{0}$ indeterminate, so fuss...

$=\lim\limits_{x\to a}\dfrac{x-a}{(x-a)(x^2+ax+a^2)}$

$=\lim\limits_{x\to a}\dfrac{1}{x^2+ax+a^2}=\dfrac{1}{a^2+a^2+a^2}=\boxed{\dfrac{1}{3a^2}}$

3. When is the graph of $y=x^4-3x^2+5x-2$ concave down?

$y'=4x^3-6x+5$

$y''=12x^2-6$

$0=6(2x^2-1)$

$x^2=\dfrac{1}{2}$

$x=\pm\sqrt{\dfrac{1}{2}}$

$\approx\pm 0.7$

y'': $+$ $-\sqrt{\frac{1}{2}}$ $-$ $\sqrt{\frac{1}{2}}$ $+$

-1 0 1

Concave down on $\left(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$

4. Determine the value of m that makes the following function continuous

$$f(x)=\begin{cases} mx^2+2 & \text{for } 0\le x\le 2 \\ x+8 & \text{for } 2<x\le 4\end{cases}$$

Must be connected at $x=2$

$m(2)^2+2=2+8$

$4m=8$

$\boxed{m=2}$

5. Given $y^2=2+xy$, find all points (x,y) on the curve where the tangent line has a slope of ½.

$2y\frac{dy}{dx}=0+x\frac{dy}{dx}+(1)y$

$\frac{dy}{dx}(2y-x)=y$

$\frac{dy}{dx}=\dfrac{y}{2y-x}$

$\frac{1}{2}=\dfrac{y}{2y-x}$

$2y-x=2y$

$-x=0$

$x=0$

$y^2=2+(0)y$

$y=\pm\sqrt{2}$

$\boxed{(0,\sqrt{2})\ (0,-\sqrt{2})}$

6. Find y' given $y = e^{2x^2+1}$

$y' = 4x\,e^{2x^2+1}$

7. Find the derivative of $f(x) = \ln\dfrac{\sqrt{2x-1}}{x(x+2)^3}$

$f(x) = \frac{1}{2}\ln(2x-1) - \ln x - 3\ln(x+2)$

$f'(x) = \dfrac{\cancel{2}\,1}{\cancel{2}(2x-1)} - \dfrac{1}{x} - \dfrac{3}{x+1}$

OK to leave as 3 separate fractions, but extra credit on test for correctly putting them together:

$f'(x) = \dfrac{x(x+1) - (2x-1)(x+1) - 3x(2x-1)}{x(x+1)(2x-1)}$

$f'(x) = \dfrac{x^2+x-(2x^2+x-1)-6x^2+3x}{x(x+1)(2x-1)}$

$f'(x) = \dfrac{-7x^2+3x+1}{x(x+1)(2x-1)}$

8. Find the derivative of $y = (2x)^{2x}$

$\ln y = [2x][\ln(2x)]$ Now product rule!

$\cancel{y}\cdot\frac{1}{\cancel{y}}\frac{dy}{dx} = \left[2\cancel{x}\cdot\frac{2}{2\cancel{x}} + 2\ln(2x)\right]y$

$\frac{dy}{dx} = (2x)^{2x}(2 + 2\ln(2x))$

9. Find the following limit: $\lim\limits_{x\to 0} x^{\ln(x+1)}$ $= 0^{\ln 1} = 0^0$ Indeterminate

Let $y = \lim\limits_{x\to 0}\left[x^{\ln(x+1)}\right]$

$\ln y = \lim\limits_{x\to 0}\left[\ln(x+1)\cdot\ln x\right]$

$\ln y = \lim\limits_{x\to 0}\left[\dfrac{\ln(x+1)}{\frac{1}{\ln x}}\right] = \dfrac{0}{\frac{1}{-\infty}} = \dfrac{0}{0}$ so L'Hop.R.

$\ln y = \lim\limits_{x\to 0}\left[\dfrac{\frac{1}{x}}{\frac{1}{\left(\frac{1}{x}\right)}}\right] = \lim\limits_{x\to 0}\left[\dfrac{1}{x^2}\right] = \dfrac{1}{0} = \boxed{\infty}$

10. Find the following limit:

$\lim\limits_{x\to -1}\left(\dfrac{x}{x^2-1} - \dfrac{1}{x+1}\right) = \dfrac{-1}{0} - \dfrac{1}{0}$

$= -\infty - \infty$

$= \boxed{-\infty}$

* $\infty - \infty$ is indeterminate, but $-\infty - \infty = -\infty$